Week 2 - Monday

# COMP 2230

# Last time

- Course overview
- Review of propositional logic
  - And
  - Or
  - Implication

# Questions?

# Assignment 1

# Logical warmup

- A New Yorker loves falafel and has two favorite places for lunch: one in Brooklyn and one in the Bronx.
- To visit the falafel place in Brooklyn he takes a train on the downtown side of the platform. To visit the falafel place in the Bronx he takes a train on the uptown side of the same platform.
- Since he likes both restaurants equally well, he simply takes the first train that comes along. In this way, he lets chance determine whether he rides to the Bronx or to Brooklyn.
- The man reaches the subway platform at a random moment around lunchtime. Brooklyn and Bronx trains arrive at the station equally often: every 10 minutes.
- Yet for some obscure reason he finds himself spending most of his time at the falafel place in Brooklyn. In fact, on the average, he goes there nine times out of 10.
- Why do the odds so heavily favor Brooklyn?

# Back to Implications

# Necessary and sufficient

- One note about implications and wording them:
  - $p$ is a **sufficient condition** for $q$ means $p \to q$
  - $p$ is a **necessary condition** for $q$ means $q \to p$
- This nomenclature is a touch counterintuitive
- Think of it this way:
  - $p \to q$ means that $p$ is enough to get you $q$, but there might be other things that will get you $q$
  - $q \to p$ means that, since you automatically get $p$ when you've got $q$, there's no way to have $q$ without $p$

# Why is implication used that way?

- $p \rightarrow q$ is true when:
  - $p$ is true and $q$ is true
  - $p$ is false
- Why?
- For the whole implication to be true, the conclusion **must** always be true when the hypothesis is true
- If the hypothesis is false, it doesn't matter what the conclusion is
- "If I punch the tooth fairy in the face, I will be Emperor of the World"
- What's the negation of an implication?

# Contrapositive

- Given a conditional statement $\boldsymbol{p} \rightarrow \boldsymbol{q}$, its **contrapositive** is $\sim\boldsymbol{q} \rightarrow \sim\boldsymbol{p}$
- Conditional:

  "If a murderer cuts off my head, then I will be dead."
- Contrapositive:

  "If I am not dead, then a murderer did not cut off my head."
- What's the relationship between a conditional and its contrapositive?

# Converse and inverse

- Given a conditional statement $p \to q$:
- Its **converse** is $q \to p$
- Its **inverse** is $\sim p \to \sim q$
- Consider the statement:
  - "If angry ham sandwiches explode, Timothée Chalamet will become immortal."
- What is its converse?
- What is its inverse?
- How are they related?

# Biconditional

- Sometimes people say "if and only if", as in:
  - "A number is prime if and only if it is divisible only by itself and 1."
- This can be written ***p*** iff ***q*** or $p \leftrightarrow q$
- This is called the biconditional and has this truth table:
- What is the biconditional logically equivalent to?

| $p$ | $q$ | $p \leftrightarrow q$ |
|---|---|---|
| T | T | T |
| T | F | F |
| F | T | F |
| F | F | T |

# Laws of Boolean algebra

| Name | Law | Dual |
|---|---|---|
| Commutative | $p \wedge q \equiv q \wedge p$ | $p \vee q \equiv q \vee p$ |
| Associative | $(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$ | $(p \vee q) \vee r \equiv p \vee (q \vee r)$ |
| Distributive | $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$ | $p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$ |
| Identity | $p \wedge \mathbf{t} \equiv p$ | $p \vee \mathbf{c} \equiv p$ |
| Negation | $p \vee {\sim}p \equiv \mathbf{t}$ | $p \wedge {\sim}p \equiv \mathbf{c}$ |
| Double Negative | ${\sim}({\sim}p) \equiv p$ | |
| Idempotent | $p \wedge p \equiv p$ | $p \vee p \equiv p$ |
| Universal Bound | $p \vee \mathbf{t} \equiv \mathbf{t}$ | $p \wedge \mathbf{c} \equiv \mathrm{c}$ |
| De Morgan's | ${\sim}(p \wedge q) \equiv {\sim}p \vee {\sim}q$ | ${\sim}(p \vee q) \equiv {\sim}p \wedge {\sim}q$ |
| Absorption | $p \vee (p \wedge q) \equiv p$ | $p \wedge (p \vee q) \equiv p$ |
| Negations of **t** and **c** | ${\sim}\mathbf{t} \equiv \mathbf{c}$ | ${\sim}\mathbf{c} \equiv \mathbf{t}$ |

# Arguments

# Arguments

- An argument is a list of statements (called **premises**) followed by a single statement (called a **conclusion**)
- Whenever all of the premises are true, the conclusion must also be true, in order to make the argument valid

# Examples

- Are the following arguments valid?
  - $p \to q \vee \sim r$ (premise)
  - $q \to p \wedge r$ (premise)
  - $\therefore p \to q$ (conclusion)

  - $p \vee (q \vee r)$ (premise)
  - $\sim r$ (premise)
  - $\therefore p \vee q$ (conclusion)

# Common argument tools

- **Modus ponens** is a valid argument of the following form:
  - $p \to q$
  - $p$
  - $\therefore q$
- **Modus tollens** is a contrapositive reworking of the argument, which is also valid:
  - $p \to q$
  - $\sim q$
  - $\therefore \sim p$
- Give verbal examples of each
- We call these short valid arguments **rules of inference**

# Generalization

- The following are also valid rules of inference:
  - $\boldsymbol{p}$
  - $\therefore \boldsymbol{p} \vee \boldsymbol{q}$

  - $\boldsymbol{q}$
  - $\therefore \boldsymbol{p} \vee \boldsymbol{q}$
- English example: "If pigs can fly, then pigs can fly or swans can breakdance."

# Specialization

- The following are also valid rules of inference:
  - $p \wedge q$
  - $\therefore p$

  - $p \wedge q$
  - $\therefore q$
- English example: "If the beat is out of control and the bassline just won't stop, then the beat is out of control."

# Conjunction

- The following is also a valid rule of inference:
  - $\boldsymbol{p}$
  - $\boldsymbol{q}$
  - $\therefore \boldsymbol{p} \land \boldsymbol{q}$
- English example: "If the beat is out of control and the bassline just won't stop, then the beat is out of control and the bassline just won't stop."

# Elimination

- The following are also valid rules of inference:
  - $p \vee q$
  - $\sim q$
  - $\therefore p$

  - $p \vee q$
  - $\sim p$
  - $\therefore q$
- English example: "If you're playing it cool or I'm maxing and relaxing, and you're not playing it cool, then I'm maxing and relaxing."

# Transitivity

- The following is also a valid rule of inference:
  - $p \to q$
  - $q \to r$
  - $\therefore p \to r$
- English example: "If you call my mom ugly I will call my brother, and if I call my brother he will beat you up, then if you call my mom ugly my brother will beat you up."

# Division into cases

- The following is also a valid rule of inference:
  - $p \lor q$
  - $p \to r$
  - $q \to r$
  - $\therefore r$
- English example: "If am fat or sassy, and being fat implies that I will give you trouble, and being sassy implies that I will give you trouble, then I will give you trouble."

# Contradiction Rule

- The following is also a valid rule of inference:
  - $\sim\boldsymbol{p} \rightarrow \mathbf{c}$
  - $\therefore \boldsymbol{p}$
- English example: "If my water is at absolute zero then the universe does not exist, thus my water must not be at absolute zero."

# Fallacies

- A **fallacy** is an argument that is not valid
- It could mean that the conclusion is not true in only a single case in the truth table
- But if the conclusion is ever false whenever all the premises are true, the argument is a fallacy
- Most arguments presented by politicians are fallacies for one reason or another

# Common fallacies

- Converse error
  - If Joe sings a sad song, then Joe will make it better.
  - Joe makes it better.
  - Conclusion: Joe sings a sad song. **FALLACY**
- Inverse error
  - If you eat too much, you will get sick.
  - You are not eating too much.
  - Conclusion: You will not get sick. **FALLACY**

# Argument practice

- First, convert the following statements into symbolic logic
  a) If this house is next to a lake, then the treasure is not in the kitchen.
  b) If the tree in the front yard is an elm, then the treasure is in the kitchen.
  c) This house is next to a lake.
  d) The tree in the front yard is an elm or the treasure is buried under the flagpole.
  e) If the tree in the back yard is an oak, then the treasure is in the garage.
- Then, apply appropriate rules of inference to determine where the treasure is hidden

# More argument practice

- First, convert the following statements about the murder of Lord Hazelton into symbolic logic
  a) Lord Hazelton was killed by a blow on the head with a brass candlestick.
  b) Either Lady Hazelton or a maid, Sara, was in the dining room at the time of the murder.
  c) If the cook was in the kitchen at the time of the murder, then the butler killed Lord Hazelton with a fatal dose of strychnine
  d) If Lady Hazelton was in the dining room at the time of the murder, then the chauffeur killed Lord Hazelton.
  e) If the cook was not in the kitchen at the time of the murder, then Sara was not in the dining room when the murder was committed.
  f) If Sara was in the dining room at the time the murder was committed, then the wine steward killed Lord Hazelton.
- Then, apply appropriate rules of inference to determine who the murderer is (if possible), assuming only one cause of death

# Upcoming

# Next time…

- Predicate logic
  - Universal quantifier
  - Existential quantifier
- Negated quantifiers
- Multiple quantifiers

# Reminders

- Read Sections 3.1, 3.2, and 3.3
- Start Assignment 1